Branch points in the complex plane and information loss in quantum systems at high level density

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Abstract

The mechanism of avoided level crossings in quantum systems is studied. It is traced back to the existence of branch points in the complex plane which influence the properties of resonance states as well as of discrete states. An avoided level crossing of two states causes not only an exchange of the two wave functions but, above all, correlations between them. The correlations play an important role at high level density since they cause the loss of information on the individual properties of the states.

I. INTRODUCTION

Recently, the generic properties of many-body quantum systems are studied with a renewed interest. They are expressed mostly by their statistical properties, i.e. by comparing their level distributions with those following from random matrix ensembles. The generic properties are, as a rule, well expressed in the center of the spectra where the level density is high. As an example, the statistical properties of the shell-model states of nuclei around ^{24}Mg are studied a few years ago [1] by using two-body forces which are obtained by fitting the low-lying states of different nuclei of the $2s-1d$ shell. In the center of the spectra, the generic properties are well expressed in spite of the two-body character of the forces used in the calculations.

A surprising result is obtained recently in performing shell-model calculations for the same systems with random two-body forces. In spite of the random character of the forces, the regular properties of the low-lying states are described quite well [2] in these calculations. Further studies [3,4] proved the relevance of the results obtained and could explain in detail even the regular properties at the border of the spectra obtained from random two-body forces [5]. Thus, generic properties of the spectra may arise from two-body forces, while regular properties may be described by means of random forces. This contra-intuitive result is, at present, not fully understood.

The spectra of microwave cavities are not determined by two-body forces. Nevertheless, the calculated spectra are similar to those from nuclear reactions [6]. They show deviations from the spectra obtained from random matrix theory as well as similarities with them. The theoretical results obtained are confirmed by experimental studies [7]. Most interesting

are, however, the results of an analysis of the statistical properties of the city transport in Cuernavaca (Mexico). It is shown in [8] that the subsequent bus arrivals display probability distributions conforming to those given by the unitary ensemble of random matrices.

By all these studies, the question on the origin of the generic properties of many-body quantum systems, observed at high level density, is raised anew. As a result of these studies, neither two-body forces nor quantum effects are decisive. The study of the city transport in Mexico can be understood as an equilibrium state of an interacting one-dimensional gas under the assumption that the information contained in the positions of the individual gas particles is minimized [8]. The minimum information is caused obviously by some information exchange with the environment.

Quantum systems having discrete states seem to exist isolated from an environment. It is therefore an interesting question whether or not some information exchange with the environment can, nevertheless, take place and provide a certain loss of information on the individual properties of the states.

The effect of avoided level crossing (Landau-Zener effect) is known and studied theoretically as well as experimentally for many years. It is a quite general property of the discrete states of a quantum system whose energies will never cross when there is a certain non-vanishing interaction between them. Instead, they avoid crossing in energy and their wave functions are exchanged when traced as a function of a certain tuning parameter. The avoided level crossings are related to the existence of exceptional points [9]. The relation of these points to geometrical phases is studied experimentally [10] as well as theoretically [11,12] in a microwave resonator by deforming it cyclically. The results show non-trivial phase changes when degeneracies appear near to one another [12]. The influence of level crossings in the complex plane onto the spectra of atoms is studied in [13]. In this case, the crossings are called hidden crossings [14].

Usually, it is assumed that avoided level crossings do not introduce any correlations between the wave functions of the states as long as the system parameter is different from the critical one at which the two states avoided crossing. Counter-examples have been found, however, in recent numerical studies of the spectra of microwave cavities [6,15]. The introduced correlations are traced back to branch points in the complex plane [16] which are the crossing points of the states when continued into the complex plane.

If it is really a general phenomenon that branch points in the complex plane introduce correlations between discrete states of a closed quantum system, then an information exchange with the environment would take place also in quantum systems. This fact could support the analogy of the statistical properties of the city transport to those of quantum systems and, above all, the interpretation of the statistical properties given in [8].

It is the aim of the present paper to study in detail the correlations in a quantum system which are introduced by branch points in the complex plane. First, the properties of the states of an open quantum system in the very neighbourhood of a branch point will be studied. In such a case, the S matrix has a double pole and the interaction between the two states is maximum. The double pole coincides with the branch point in the complex plane. It is shown further, that the branch point influences the properties of the resonance states not only when the physical conditions for a double pole of the S matrix are fulfilled but also when the states avoid crossing. This result corresponds to those obtained for the realistic case of laser induced continuum structures in atoms [17]. The influence of the branch points

is accompanied, in any case, by an exchange of the wave functions of the states.

The properties of resonance states in the neighbourhood of an avoided crossing are traced in the further studies of the present paper. More exactly: starting from the branch point where two resonance states with given widths $\gamma_1^{\rm cr}$ and $\gamma_2^{\rm cr}$ cross, the properties of two other states with γ_i smaller or larger $\gamma_i^{\rm cr}$ are studied. For these two states, the physical conditions for the formation of a double pole of the S matrix are not fulfilled. Finally, the γ_i are chosen to be zero. This study allows to compare the properties of resonance states with finite lifetime with those of discrete states under the influence of a branch point in the complex plane. The results show how the branch points in the complex plane continue analytically into the function space of discrete states.

The paper is organized as follows. In section 2, the mathematical properties of branch points in the complex plane and their relation to avoided level crossings are sketched. In section 3, numerical results for states at a double pole of the S matrix as well as for states with avoided crossing are given. The main point studied are the correlations of the wave functions introduced by an avoided level crossing. The results are discussed in the last section. They show that some information loss appears in quantum systems at high level density under the influence of the branch points in the complex plane. The information contained in the positions and wave functions of the individual states of the system is minimized in a similar manner as in the city transport in Mexico.

II. BRANCH POINTS IN THE COMPLEX PLANE

The avoided crossing of discrete levels is related to the existence of branch points in the complex plane. In order to illustrate this relation, let us study the properties of the complex two-by-two Hamiltonian matrix

$$\mathcal{H} = \begin{pmatrix} e_1(a) & 0 \\ 0 & e_2(a) \end{pmatrix} - \begin{pmatrix} \frac{i}{2}\gamma_1(a) & \omega \\ \omega & \frac{i}{2}\gamma_2(a) \end{pmatrix} . \tag{1}$$

The unperturbed energies e_i and widths γ_i $(i = 1, 2)$ of the two states depend on the parameter a to be tuned in such a manner that the two states may cross in energy (and/or width) when $\omega = 0$. The two states interact only via the non-diagonal matrix elements ω which may be complex, in general. The imaginary part of ω arises from the residuum of the integral describing the interaction via the continuum of decay channels, see e.g. [18,19]. The real part contains both the direct interaction of the two states and the principal value integral which appears from the interaction via the continuum. In the following, we consider only real ω (and γ_i independent of a) since we are interested in the influence of the branch points onto the avoided crossing of discrete states. The influence of complex ω on the crossing of resonance states will be considered in a forthcoming paper.

The eigenvalues of $\mathcal{H}$ are

$$\mathcal{E}_\pm \equiv E_\pm - \frac{i}{2}\Gamma_\pm = \frac{\epsilon_1 + \epsilon_2}{2} \pm \frac{1}{2}\sqrt{(\epsilon_1 - \epsilon_2)^2 + 4\omega^2} \tag{2}$$

with $\epsilon_j \equiv e_j - \frac{i}{2}\,\gamma_j$ $\;(j = 1, 2)$. According to equation (2), two interacting discrete states (with $\gamma_i = 0$) avoid always crossing since ω is real in this case. It is, however, possible to find

the corresponding crossing point in the complex plane by considering non-vanishing values g_i instead of the γ_i. Both, the g_i and the γ_i, are proportional to the spectroscopic factors [16]. The common energy dependent factor in the γ_i causes $\gamma_i \to 0$ in approaching the decay threshold where the resonance states become discrete. The common energy dependent factor in the g_i is different from zero also for discrete states. The g_i describe the transfer probability between different states.

It can further be seen from equation (2) that these crossing points are branch points in the complex plane. Equation (2) shows also that resonance states with non-vanishing widths γ_i avoid mostly crossing since $F(a,\omega) \equiv (\epsilon_1 - \epsilon_2)^2 + 4\omega^2$ is different from zero for all a, as a rule. Only when $F(a,\omega) = 0$ at $a = a^{\rm cr}$ (and $\omega = \omega^{\rm cr}$), the states cross. In such a case, the S matrix has a double pole, see e.g. [20]. Note that the branch point is determined by the values $(\omega)^2$ and $(\epsilon_1 - \epsilon_2)^2$ but not by the signs of these values. According to equation (2), it lies at $X \equiv (1/2)\{\epsilon_1(a^{\rm cr}) + \epsilon_2(a^{\rm cr})\}$.

The left and right eigenfunctions, $\Phi_i^{\rm lt}$ and $\Phi_i^{\rm rt}$ ($i = \pm$), of a non-Hermitian matrix are different from one another. Since the matrix (1) is symmetrical, it follows

$$\langle \Phi_i^* | \, \mathcal{H} = \langle \Phi_i^* | \, \mathcal{E}_i \qquad \text{and} \qquad \mathcal{H} \, |\Phi_i\rangle = \mathcal{E}_i \, |\Phi_i\rangle \; , \tag{3}$$

see e.g. [17,19,21]. Therefore, $\Phi_i^{\rm lt} = \Phi_i^{\rm rt\,*} \equiv \Phi_i^*$. The eigenfunctions of $\mathcal{H}$ can be orthonormalized according to

$$\langle \Phi_i^{\rm lt} | \Phi_j^{\rm rt} \rangle = \langle \Phi_i^* | \Phi_j \rangle = \delta_{ij} \tag{4}$$

where $\Phi_j^{\rm rt} \equiv \Phi_j$. Equation (4) provides the bi-orthogonality relations

$$\begin{aligned} \langle \Phi_i | \Phi_i \rangle &= \Re(\langle \Phi_i | \Phi_i \rangle) = \langle \Phi_j | \Phi_j \rangle \;\; ; \;\; A \equiv \langle \Phi_i | \Phi_i \rangle \ge 1 \\ \langle \Phi_i | \Phi_{j\ne i} \rangle &= i \, \Im(\langle \Phi_i | \Phi_{j\ne i} \rangle) = -\langle \Phi_{j\ne i} | \Phi_i \rangle \;\; ; \;\; B \equiv |\langle \Phi_i | \Phi_{j\ne i} \rangle| \ge 0 \; . \end{aligned} \tag{5}$$

At the double pole, $|\langle \Phi_i | \Phi_i \rangle| \to \infty$ and $|\langle \Phi_i | \Phi_{j\ne i} \rangle| \to \infty$ but $\langle \Phi_i^* | \Phi_j \rangle = \delta_{ij}$ according to equation (4). Numerical examples for the values $\langle \Phi_i | \Phi_j \rangle$ with $i = j$ as well as with $i \ne j$ can be found in [19,21–23]. Further, at the double pole the two wave functions are related by $\Phi_i = \pm \, i \, \Phi_{j\ne i}$ [17].

The eigenfunctions Φ_i can be represented in the set of basic wave functions Φ_i^0 of the unperturbed matrix corresponding to $\omega = 0$,

$$\Phi_i = \sum b_{ij} \Phi_j^0 \; . \tag{6}$$

In the critical region of avoided crossing, the eigenfunctions are mixed: $b_{ii} = b_{jj}$ and $b_{ij} = -b_{ji}$ for $i \ne j$. The b_{ij} are normalized according to equation (4).

III. NUMERICAL RESULTS

The numerical results obtained by diagonalizing the matrix (1) are shown in figures 1 to 6. In all cases $e_1 = 1 - a/2$, $e_2 = a$ and $\omega = 0.05$. The γ_i do not depend on the tuning parameter a. At $a = a^{\rm cr} = 2/3$, the two levels cross when unperturbed (i.e. $\omega = 0$) and avoid crossing, as a rule, when perturbed by the interaction ω. When $\gamma_1/2 = \gamma_1^{\rm cr}/2 = 1.0$, $\gamma_2/2 =$

$\gamma_2^{\rm cr}/2 = 1.1$, the S matrix has a double pole meaning that the two resonance states cross in spite of $\omega \neq 0$. According to equation (2), the double pole of the S matrix is a branch point in the complex plane.

In figure 1, the energies $E_\pm$, widths $\Gamma_\pm$ and wave functions b_{ij} of the two states are shown as a function of the parameter a in the very neighbourhood of the branch point. Approaching the branch point at $a^{\rm cr}$, $|\Re(b_{ij})| \to \infty$ and $|\Im(b_{ij})| \to \infty$. While $\Re(b_{ij})$ does not change its sign by crossing the critical value $a^{\rm cr}$, the phase of $\Im(b_{ij})$ jumps from $\pm$ to $\mp$. The orthogonality relations (4) are fulfilled for all a including the critical value $a^{\rm cr}$.

Figure 2 shows the energies $E_\pm$ and widths $\Gamma_\pm$ of the two states for values of γ_i just above and below the critical ones $\gamma_i^{\rm cr}$ and for $\gamma_i = 0$, i.e. for discrete states. When $\gamma_i > \gamma_i^{\rm cr}$, the widths of the two states approach each other near $a^{\rm cr}$ but the width of one of the states remains always larger than the width of the other one. The two states cross freely in energy, and the wave functions are not exchanged after crossing the critical value $a^{\rm cr}$.

The situation is completely different when $\gamma_i < \gamma_i^{\rm cr}$. In this case, the states avoid crossing in energy while their widths cross freely. After crossing the critical value $a^{\rm cr}$, the wave functions of the two states are exchanged. An exchange of the wave functions takes place also in the case of discrete states ($\gamma_i = 0$). This latter result is well known as Landau-Zener effect. It is directly related to the branch point in the complex plane at $a^{\rm cr}$ as can be seen from figure 2.

The wave functions b_{ij} are shown in figure 3. The states are mixed (i.e. $|b_{ii}| \neq 1$ and $b_{ij\neq i} \neq 0$) in all cases in the neighbourhood of $a^{\rm cr}$. In the case without exchange of the wave functions, $\Re(b_{ij})$ as well as $\Im(b_{ij})$ behave smoothly at $a^{\rm cr}$ while this is true only for $\Re(b_{ij})$ in the case with exchange of the wave functions. In this case, $\Im(b_{ij})$ jumps from a certain finite value y to $-y$ at $a^{\rm cr}$. The $\Im(b_{ij})$ of discrete states are zero. Thus, a jump in the $\Im(b_{ij})$ can not appear. The $\Re(b_{ij})$, however, show a dependence on a which is very similar to that of resonance states with exchange of the wave functions ($\gamma_i < \gamma_i^{\rm cr}$).

In order to trace the influence of the branch point in the complex plane onto the correlations of discrete states, the differences $\delta = |b_{ii}|^2 - |b_{ij\neq i}|^2$ and the values $|b_{ij}|^2$ are shown in figures 4 and 5 for different values γ_i from $\gamma_i > \gamma_i^{\rm cr}$ to $\gamma_i = 0$. Most interesting is the change of the value δ from 1 to 0 at $\gamma_i^{\rm cr}$. The relation $|b_{ii}|^2 = |b_{ij\neq i}|^2$ at $\gamma < \gamma_i^{\rm cr}$ is the result from interference processes. It holds also at $\gamma_i = 0$, i.e. for discrete states. In this case, $|b_{ii}|^2 = |b_{ij\neq i}|^2 = 0.5$ at $a^{\rm cr}$.

The values A and B characterizing the bi-orthogonality of the two wave functions are shown in figure 6 for the same values of γ_i as in figures 4 and 5. The A and B are similar for $\gamma_i - \gamma_i^{\rm cr} = \pm\Delta$ as long as Δ is small. They approach $A \to 1$ and $B \to 0$ for $\gamma_i \to 0$.

In figure 7, the energies E_i and mixing coefficients $|b_{ij}|^2$ are shown for illustration for four discrete states with three neighboured avoided crossings as a function of a. In analogy to (1), the matrix is

$$\mathcal{H}^{(4)} = \begin{pmatrix} e_1(a) & 0 & 0 & 0 \\ 0 & e_2(a) & 0 & 0 \\ 0 & 0 & e_3(a) & 0 \\ 0 & 0 & 0 & e_4(a) \end{pmatrix} - \begin{pmatrix} 0 & \omega_{12} & \omega_{13} & \omega_{14} \\ \omega_{21} & 0 & \omega_{23} & \omega_{24} \\ \omega_{31} & \omega_{32} & 0 & \omega_{34} \\ \omega_{41} & \omega_{42} & \omega_{43} & 0 \end{pmatrix} . \qquad (7)$$

The correlations introduced by the avoided crossings remain at all values of the parameter a at high level density. They are the result of complicated interference processes. This can

be seen best by comparing the two pictures with four interacting states (top and middle in figure 7) with those of only two interacting states (bottom of figure 7 and bottom right of figure 5). Figure 7 bottom shows the large region of the a values around $a^{\rm cr}$ for which the two wave functions remain mixed: $|b_{ii}|^2 \to 1$ and $|b_{ij\neq i}|^2 \to 0$ for $a \to a^l$ with $|a^l - a^{\rm cr}_{34}| \gg |a^{\rm cr}_{i4} - a^{\rm cr}_{34}|$; $i = 1, 2$. The avoided crossings between neighboured states do, therefore, not occur between states with pure wave functions and it is impossible to identify the $|b_{ij}|^2$ unequivocally (figure 7 right top and middle).

IV. DISCUSSION OF THE RESULTS

Most calculations represented in the present paper are performed for two states which cross or avoid crossing under the influence of an interaction ω which is real. A general feature appearing in all the results is the repulsion of the levels in energy (except in the very neighbourhood of $a^{\rm cr}$ when $\gamma_i \geq \gamma_i^{\rm cr}$). This result follows analyically from the eigenvalue equation (2). It holds quite generally for real ω as discussed by means of the spectra of microwave cavities [6] and laser-induced continuum structures in atoms [17]. The level repulsion in energy is accompanied by an approaching of the lifetimes (widths) of the states.

The value $a^{\rm cr}$ is determined by the free crossing in energy of the two non-interacting states Φ_i^0 (corresponding to $\omega = 0$) in the cases considered. The avoided level crossing at the critical value $a^{\rm cr}$ can be seen clearly only when the interaction ω is small. For larger ω, the avoided crossings are washed out and are difficult to identify without performing a detailed analysis of the data. As a consequence, there are much more avoided level crossings in quantum systems at high level density than usually believed and the influence of the branch points may be quite important. The last point does not coincide with any intuitive assumptions.

The value of the (real) interaction ω in relation to the value of the widths γ_i is decisive whether or not the states will be exchanged at the critical value $a^{\rm cr}$ of the tuning parameter, i.e. whether or not the states avoid crossing. When ω is so small that the difference of the widths $\Gamma_+ - \Gamma_-$ is different from zero at $a^{\rm cr}$ then the states will not be exchanged. If, however, $\Gamma_+ = \Gamma_-$ at $a^{\rm cr}$, the states will be exchanged. The exchange occurs due to interferences between the different components of the wave functions. This can be seen best for resonance states with $\gamma_i \neq 0$ where interferences between the real and imaginary parts of the wave functions play an important role (see figure 5).

At the double pole of the S matrix, not only $\Gamma_+ = \Gamma_-$ but also $E_+ = E_-$ for $a^{\rm cr}$. Here, the real and imaginary parts of all components of the wavefunctions increase boundless and $\Phi_i \to \pm\, i\, \Phi_{j\neq i}$ [17]. Avoided level crossings are characterized by $E_+ \neq E_-$ at $a^{\rm cr}$. In these cases, the increase of the components of the wave functions at $a^{\rm cr}$ is reduced due to interferences. The interferences can be seen, e.g., from the differences $\delta = |b_{ii}|^2 - |b_{ij\neq i}|^2$ which jump from 1 to 0 at the branch point (see figure 4). This jump is related to the normalization condition (4) which is fulfilled also at the double pole of the S matrix. The jump makes possible the exchange of the wave functions. The value $\delta = 0$ remains when $(\gamma_1/2 - \gamma_2/2)^2 < 4\,\omega^2$, i.e. also for discrete states.

The bi-orthogonality (5) of the wave functions characterizes the avoided crossing of resonance states. It increases boundless at the double pole of the S matrix and vanishes in the case of an avoided crossing of discrete states (figure 6).

The results of all the calculations presented in the present paper show very clearly that, from a mathematical point of view, the branch points in the complex plane continue analytically into the function space of discrete states. Their influence on the properties of discrete states cannot be neglected.

Another result of the present study is the influence of the branch points in the complex plane onto the purity of the wave functions $\Phi_{\pm}$. At $a^{\rm cr}$, the wave functions are not only exchanged but become mixed. The mixing occurs not only at the critical point $a^{\rm cr}$ but in a certain region around $a^{\rm cr}$ when the crossing is avoided. This fact is important at high level density where, as a rule, an avoided crossing with another level appears before $\Phi_i \to \Phi_j^0$ is reached. As a result, all the wave functions of closely-lying states are strongly mixed, i.e. the states are strongly correlated at high level density (see figure 7).

The strong mixing of the wave functions of a quantum system at high level density means that the information on the individual properties of the states Φ_i^0 is lost. While the exchange of the wave functions itself is of no interest for a statistical consideration of the states, the accompanying correlation of the states is decisive for the statistics. This fact is discussed also in [24]. As shown in the present paper, the correlation of the states may be traced back to the existence of branch points in the complex plane. The number of branch points in the complex plane is large at high level density. Therefore, the discrete states of quantum systems at high level density lose any information on their individual properties. That means, they contain minimum information.

Thus, there is an information exchange between a (closed) quantum system and the continuum due to the analyticity of the wave functions. The branch points in the complex plane are *hidden crossings*, indeed. They play an important role not only in atoms, as supposed in [13,14], but determine the properties of all quantum systems at high level density.

The loss of information on the properties of discrete states of a quantum system under the influence of branch points in the complex plane is in complete analogy to the conclusion drawn from the statistical properties of the city transport in Cuernavaca (Mexico) [8]. Also in this case, an information exchange with the environment leads to an information loss in the system.

It follows further that the statistical properties of quantum systems at high level density are different from those at low level density. States at the border of the spectrum are almost not influenced by branch points in the complex plane since their number is small. There are almost no states which could cross or avoid crossing with others states. The properties of these states are expected therefore to show more individual features than those at high level density. In other words, the information on the individual properties of the states Φ_i^0 at the border of the spectrum is kept to a great deal in contrast to that on the states in the center of the spectrum.

Summarizing the results, it can be stated the following. Branch points in the complex plane cause an exchange of the wave functions and, above all, create correlations between the states of a quantum system at high level density even if the system is closed and the states are discrete. These correlations are equivalent to a loss of information on the individual basic states. This result may explain the similarity between the statistical properties of the

city transport in Cuernavaca (Mexico) and those of a quantum system at high level density.

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FIGURES

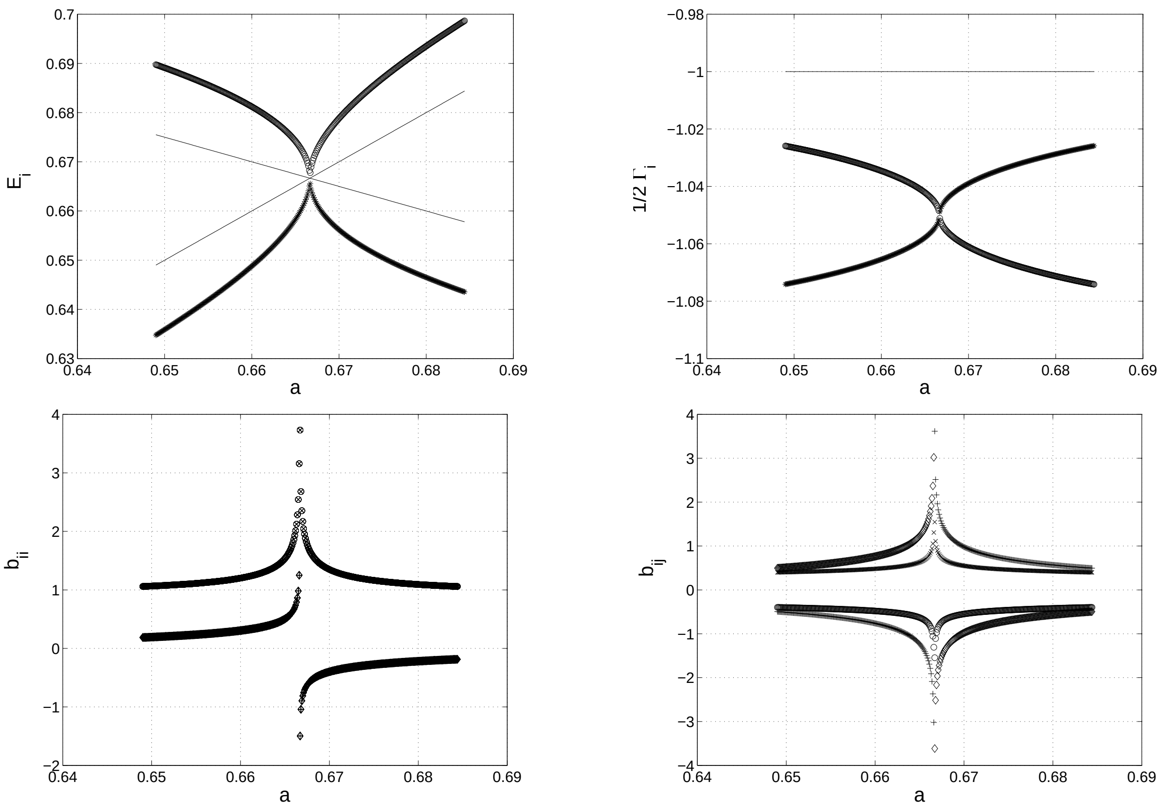

FIG. 1. The energies E_i (top left) and widths $\Gamma_i/2$ (top right) of the two eigenstates of the matrix (1) as a function of the parameter a. The thin lines give the energies E_i and widths $\Gamma_i/2$ of the states at $\omega = 0$. The lower part of the figure shows the coefficients b_{ii} (bottom left) and $b_{ij\neq i}$ (bottom right) defined by equation (6). The x and o denote the $\Re(b_{ij})$ while the $\Im(b_{ij})$ are denoted by $+$ and $\diamond$. $e_1 = 1 - a/2$; $e_2 = a$; $\gamma_1/2 = 1.0$; $\gamma_2/2 = 1.1$ and $\omega = 0.05$.

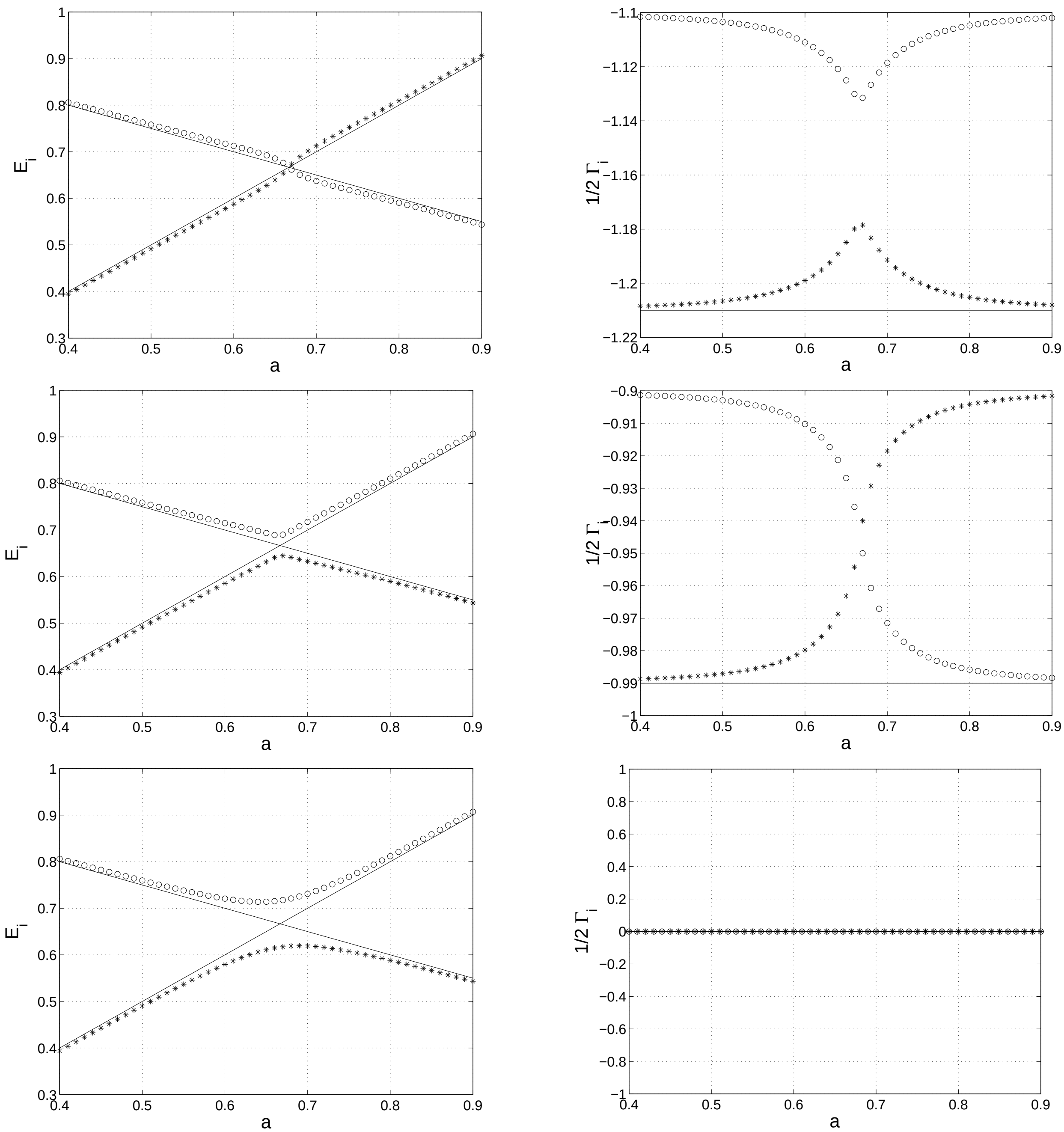

FIG. 2. The energies E_i (left) and widths $\Gamma_i/2$ (right) as a function of the tuning parameter a. $e_1 = 1 - a/2$; $e_2 = a$ and $\omega = 0.05$. The $\gamma_1/2$ are 1.10 (top), 0.90 (middle), 0 (bottom); $\gamma_2 = 1.1 \cdot \gamma_1$. The full lines show the E_i and $\Gamma_i/2$ for $\omega = 0$.

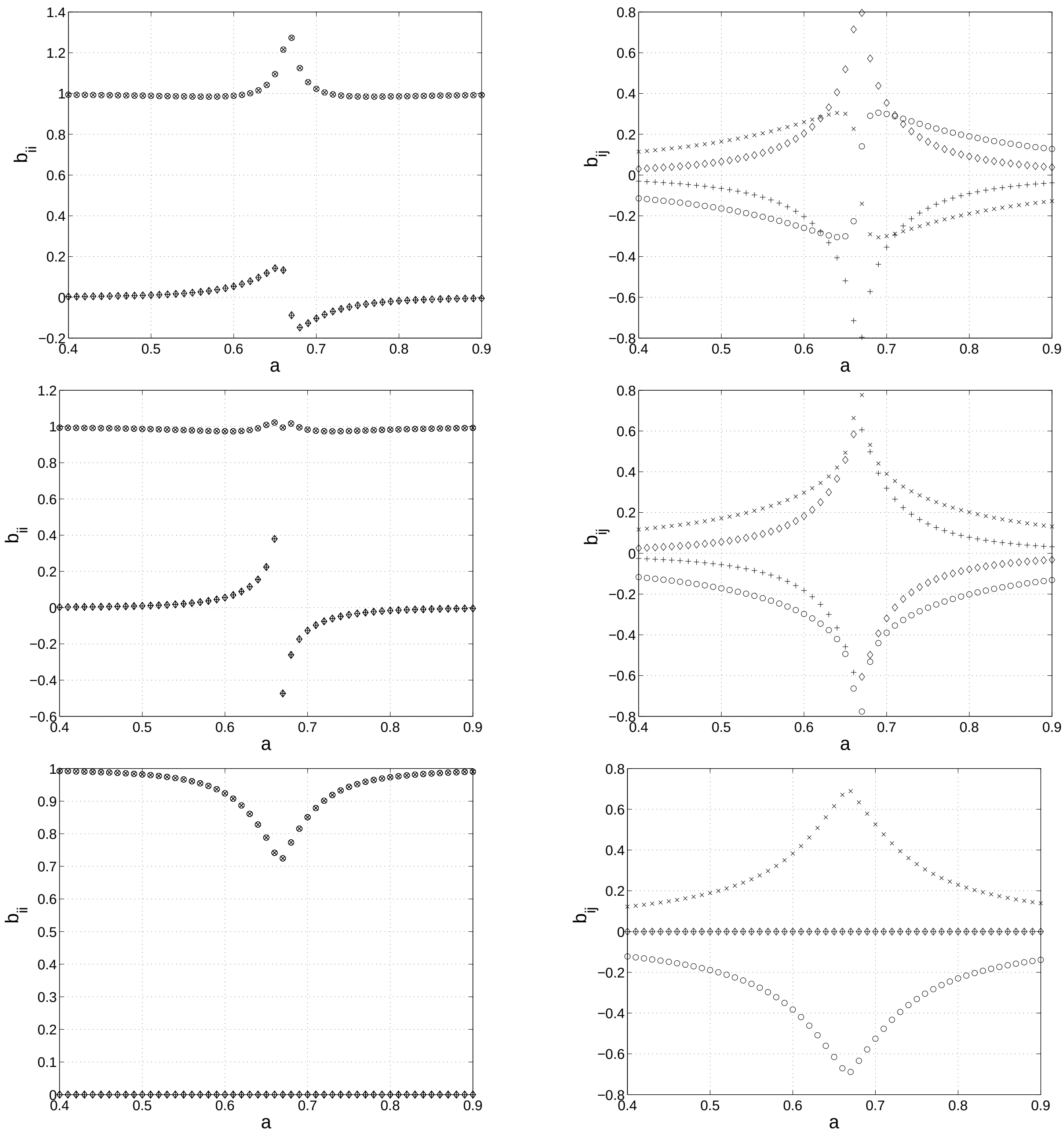

FIG. 3. The mixing coefficients b_{ii} (left) and $b_{ij\neq i}$ (right) defined by equation (6) as a function of the tuning parameter a. o and x denote the real parts and $\diamond$ and + the imaginary parts. $e_1 = 1 - a/2$; $e_2 = a$ and $\omega = 0.05$. The $\gamma_1/2$ are the same as in figure 2: 1.10 (top), 0.90 (middle), 0 (bottom); $\gamma_2 = 1.1 \cdot \gamma_1$. Note the different scales in the three cases.

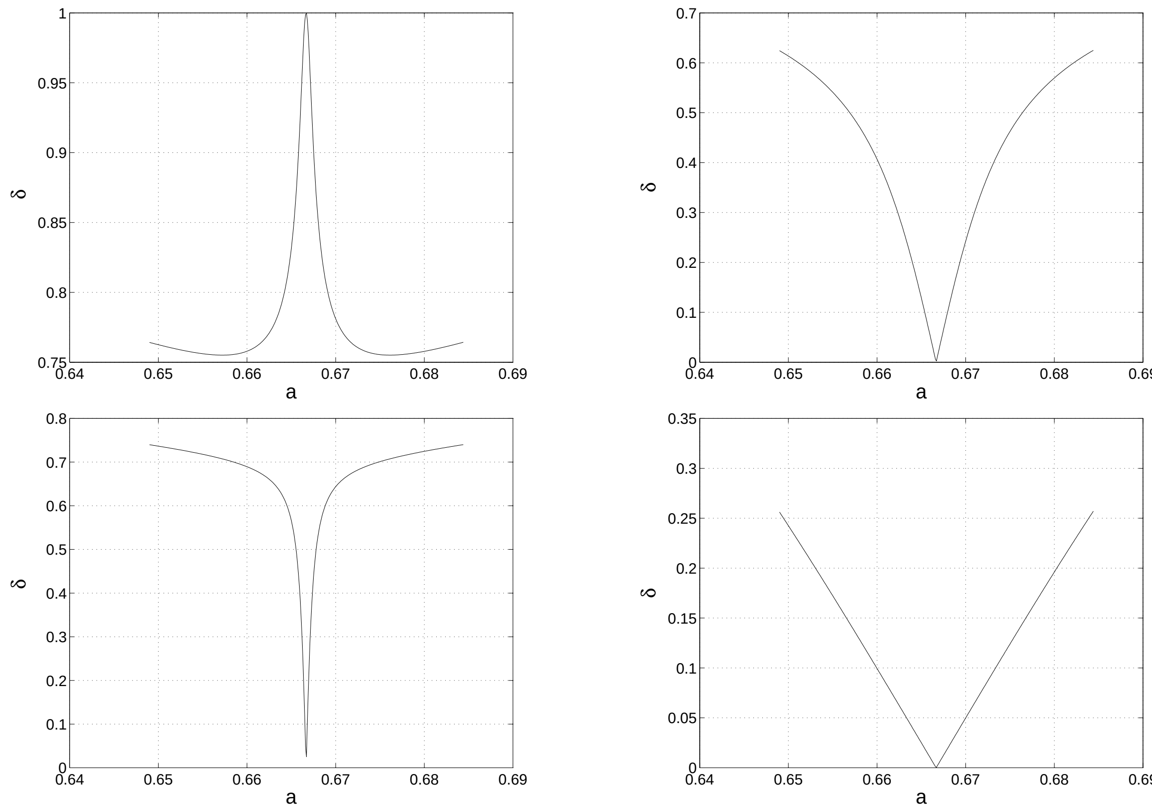

FIG. 4. The differences $\delta = |b_{ii}|^2 - |b_{ij\neq i}|^2$ as a function of the tuning parameter a. $e_1 = 1 - a/2$; $e_2 = a$ and $\omega = 0.05$. The $\gamma_1/2$ are 1.010 (top left), 0.990 (bottom left), 0.90 (top right), 0 (bottom right); $\gamma_2 = 1.1 \cdot \gamma_1$. Note the different scales in the different figures.

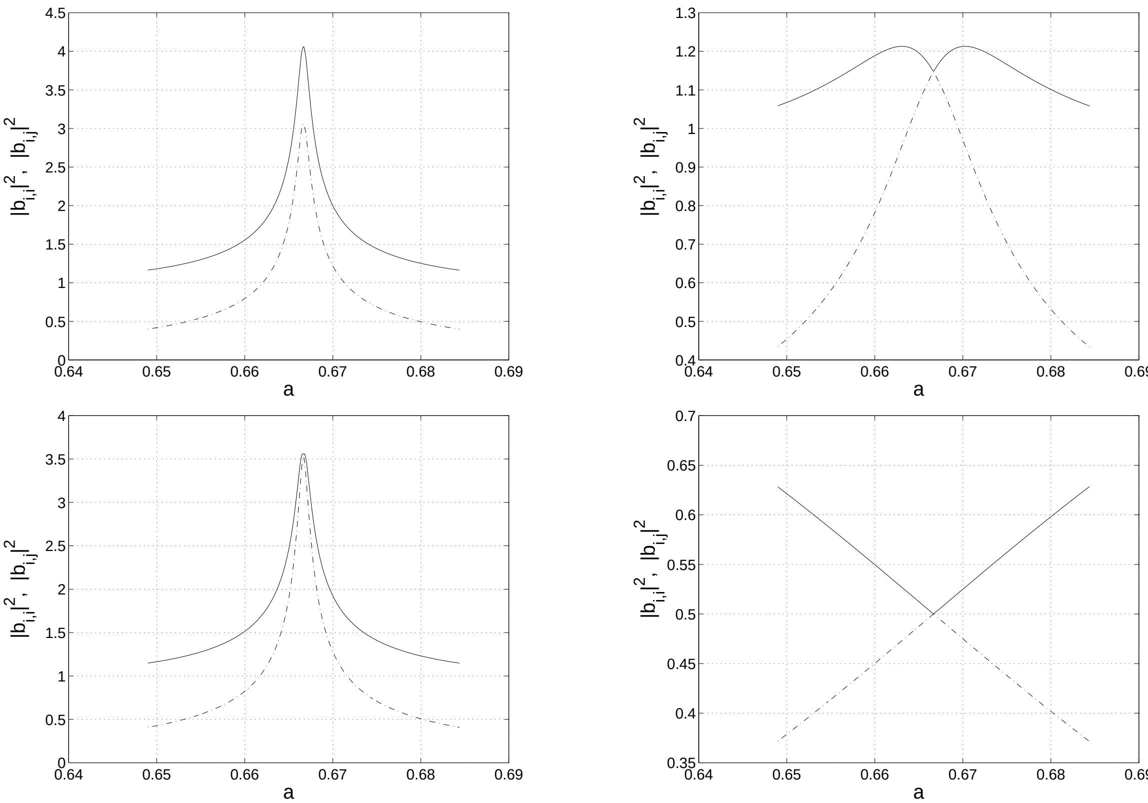

FIG. 5. The $|b_{ii}|^2$ (full lines) and $|b_{ij\neq i}|^2$ (dash-dotted lines) as a function of the tuning parameter a. $e_1 = 1 - a/2$; $e_2 = a$ and $\omega = 0.05$. The $\gamma_1/2$ are the same as in figure 4: 1.010 (top left), 0.990 (bottom left), 0.90 (top right), 0 (bottom right); $\gamma_2 = 1.1 \cdot \gamma_1$. Note the different scales in the different figures.

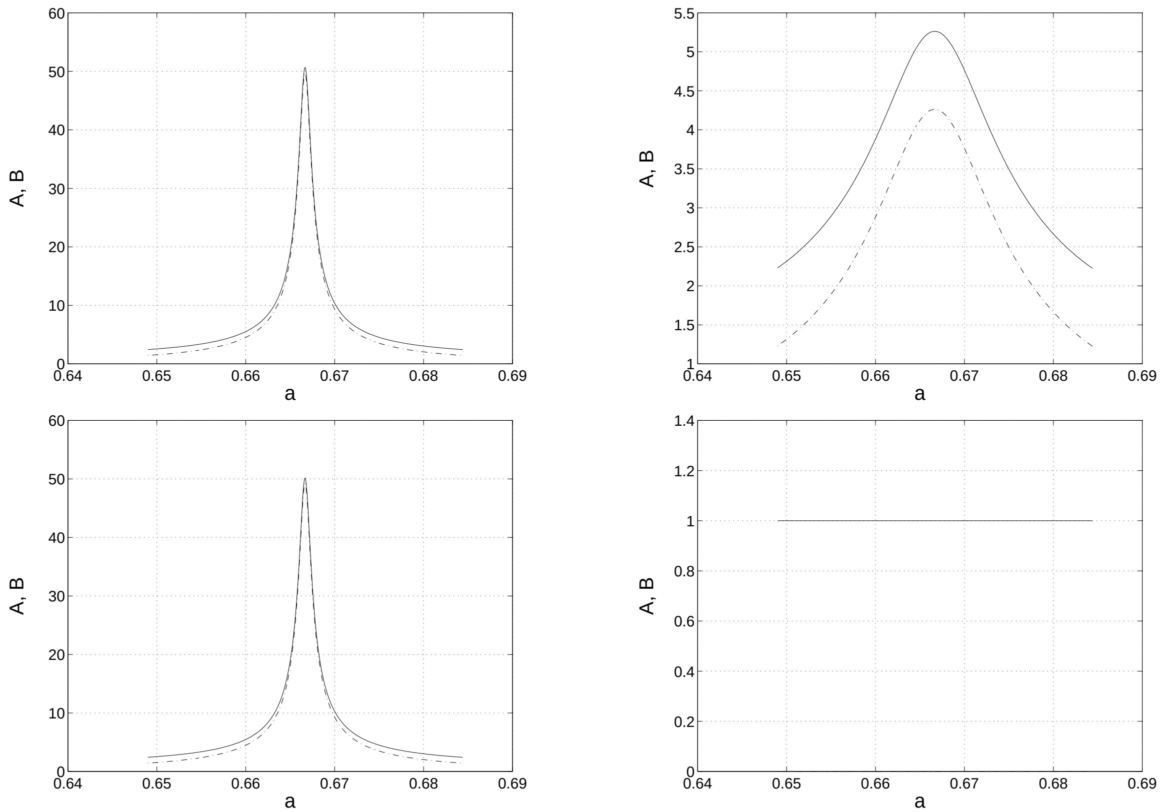

FIG. 6. The A (full lines) and B (dash-dotted lines) defined in equation (5) as a function of the tuning parameter a. $e_1 = 1 - a/2$; $e_2 = a$ and $\omega = 0.05$. The $\gamma_1/2$ are the same as in figure 4: 1.010 (top left), 0.990 (bottom left), 0.90 (top right), 0 (bottom right); $\gamma_2 = 1.1 \cdot \gamma_1$. Note the different scales in the different figures.

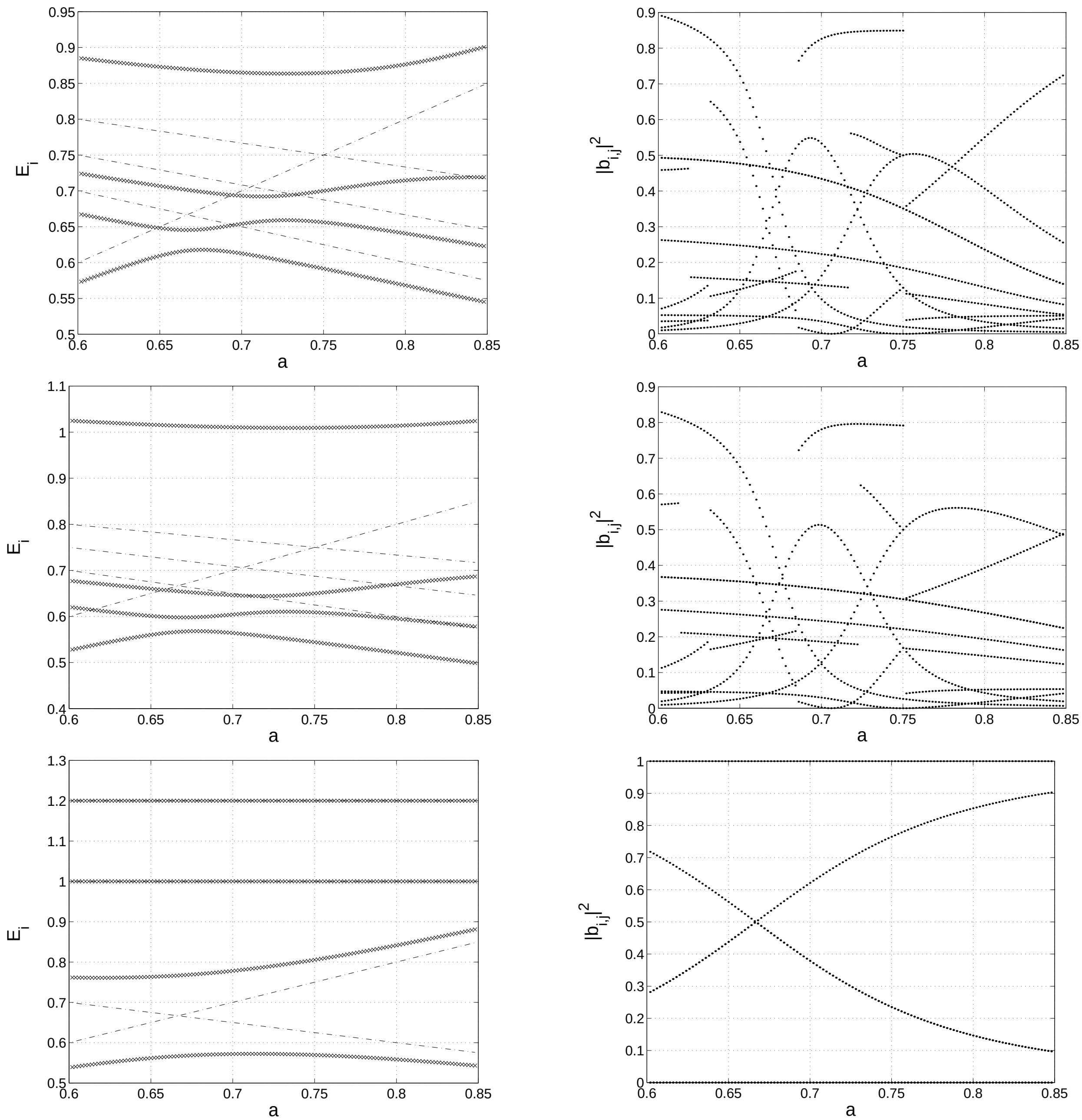

FIG. 7. The energies E_i (left) and mixing coefficients $|b_{ij}|^2$ (right) of four discrete states ($\gamma_i = 0$ for $i = 1, ..., 4$) obtained from $\mathcal{H}^{(4)}$, equation (7), as a function of the tuning parameter a. Top and middle: $e_1 = 1 - a/3$; $e_2 = 1 - 5a/12$; $e_3 = 1 - a/2$; $e_4 = a$; $\omega = 0.05$ (top) and 0.1 (middle) for all non-diagonal matrix elements. Bottom: the same as above but $e_1 = 1$; $e_2 = 1.2$; $\omega = 0$ for the coupling between the states $i = 1, 2$ and $j \neq i$, $\omega = 0.1$ for the coupling between $i = 3, 4$ and $j = 4, 3$. In this case, $|b_{ii}|^2 \geq |b_{ij \neq i}|^2$ (bottom right) as in figure 5 (bottom right). The dash-dotted lines (left) show E_i for $\omega = 0$. The states i and j are exchanged at some values a.